

A First Look at $N_f = 3$ Dynamical DWF Simulations

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1. Chiral symmetry and DWF
2. Parameters of $N_f = 0, 2$ and 3 simulations
3. m_{res} , m_ρ and m_π
4. Plaquette distributions and L_s dependence of m_{res}

Domain Wall Fermion Operator

- Introduce extra dimension, labeled by s

$$D_{x,s;x',s'} = \delta_{s,s'} D_{x,x'}^{\parallel} + \delta_{x,x'} D_{s,s'}^{\perp}$$

- $D_{x,x'}^{\parallel}$ is a Wilson Dirac operator with an opposite sign for the mass term.

$$D_{x,x'}^{\parallel} = \frac{1}{2} \sum_{\mu=1}^4 \left[(1 - \gamma_\mu) U_{x,\mu} \delta_{x+\hat{\mu},x'} + (1 + \gamma_\mu) U_{x',\mu}^\dagger \delta_{x-\hat{\mu},x'} \right] + (M_5 - 4) \delta_{x,x'}$$

- $D_{s,s'}^{\perp}$ couples nearest neighbors in the fifth dimension, distinguishing left- and right-handed fermions

$$\begin{aligned} D_{s,s'}^{\perp} &= \frac{1}{2} \left[(1 - \gamma_5) \delta_{s+1,s'} + (1 + \gamma_5) \delta_{s-1,s'} - 2 \delta_{s,s'} \right] \\ &- \frac{m_f}{2} \left[(1 - \gamma_5) \delta_{s,L_s-1} \delta_{0,s'} + (1 + \gamma_5) \delta_{s,0} \delta_{L_s-1,s'} \right] \end{aligned}$$

Residual Chiral Symmetry Breaking for DWF

- Consider introducing in action a $SU(N_f)$ matrix Ω through term at $l \equiv L_s/2$

$$-\sum_x \left\{ \bar{\Psi}_{x,l-1} P_L (\Omega^\dagger - 1) \Psi_{x,l} + \bar{\Psi}_{x,l} P_R (\Omega - 1) \Psi_{x,l-1} \right\} \quad \Omega \rightarrow U_R \Omega U_L^\dagger$$

- Conventional DWF recovered by $\Omega \rightarrow 1$

- QCD chiral Lagrangian $\mathcal{L}_{\text{QCD}}^{(2)}$, with $\Sigma \equiv \exp [2i\phi^a t^a/f]$ and mass matrix M is:

$$\frac{f^2}{8} \text{Tr} \left(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right) + v \text{Tr} \left[M \Sigma + (M \Sigma)^\dagger \right] + v' \text{Tr} \left[\Omega \Sigma + (\Omega \Sigma)^\dagger \right] + v'' \text{Tr} \left[\Omega M^\dagger + \Omega^\dagger M \right]$$

- For modes bound to walls of fifth dimension, Ω enters Green's functions as

$$\Omega e^{-\alpha L_s} \quad \Rightarrow \quad v', v'' \sim e^{-\alpha L_s}$$

- Chiral condensate from differentiating w.r.t. mass, m_π^2 from expanding Σ

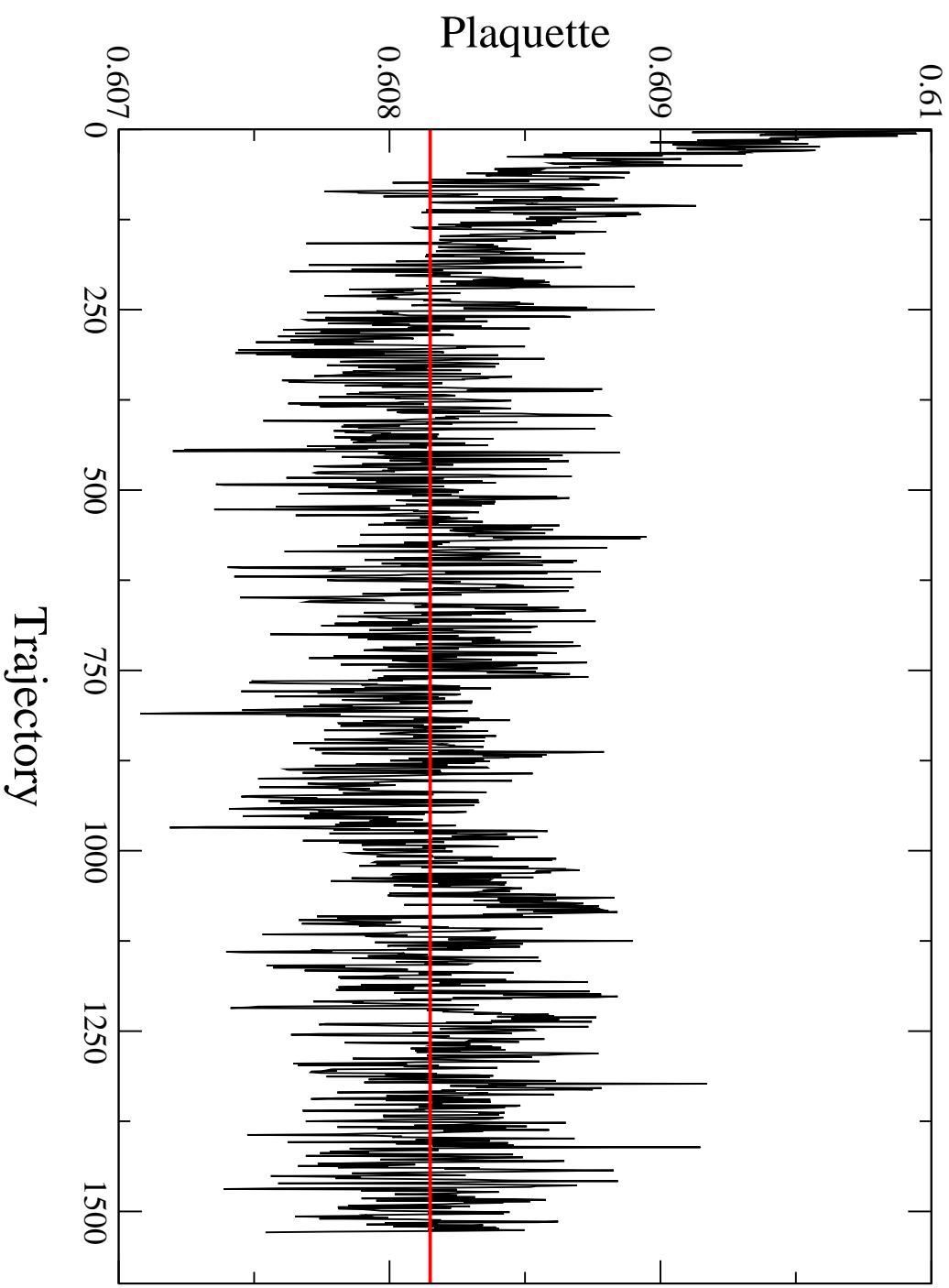
$$-\langle \bar{q}q \rangle (m_f = 0, L_s) \sim v + v'' \quad v = \frac{f^2 m_{\pi^+}^2}{4(m_u + m_d + 2m_{\text{res}})} \quad m_{\text{res}} \equiv v'/v$$

$N_f = 0$, $N_f = 2$ and $N_f = 3$ DWF Calculations

Parameter	$N_f = 0$	$N_f = 0$	$N_f = 2$	$N_f = 3$
Gauge action	Wilson	DBW2	DBW2	DBW2
β	6.0	1.04	0.80	0.72
Volume	$16^3 \times 32$	$16^3 \times 32$	$16^3 \times 32$	$16^3 \times 32$
L_s	16	16	12	8
a^{-1} (GeV)	1.92(4)	1.98(2)	1.70(5)	≈ 1.7
m_{res}	$1.24(5) \times 10^{-3}$	$1.85(12) \times 10^{-5}$	$1.37(2) \times 10^{-3}$	$1.17(1) \times 10^{-2}$
Dyn. masses	–	–	0.02, 0.03, 0.04	0.04
Algorithm	HB	HB + OR	HMC	R
Trajectories	–	–	5361 (0.02) 6195 (0.03) 5605 (0.04)	1525

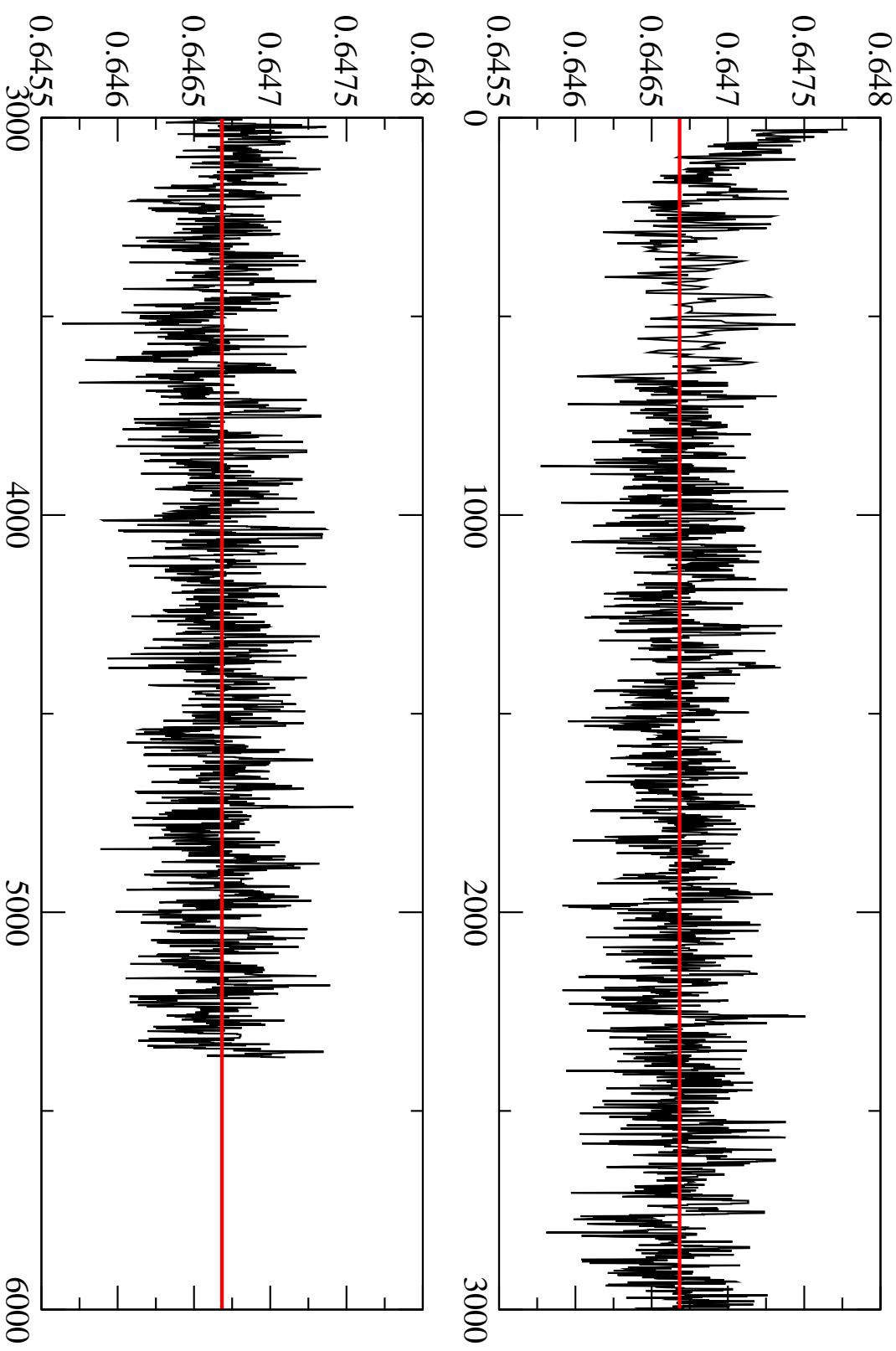
Plaquette Evolution for $N_f = 3$

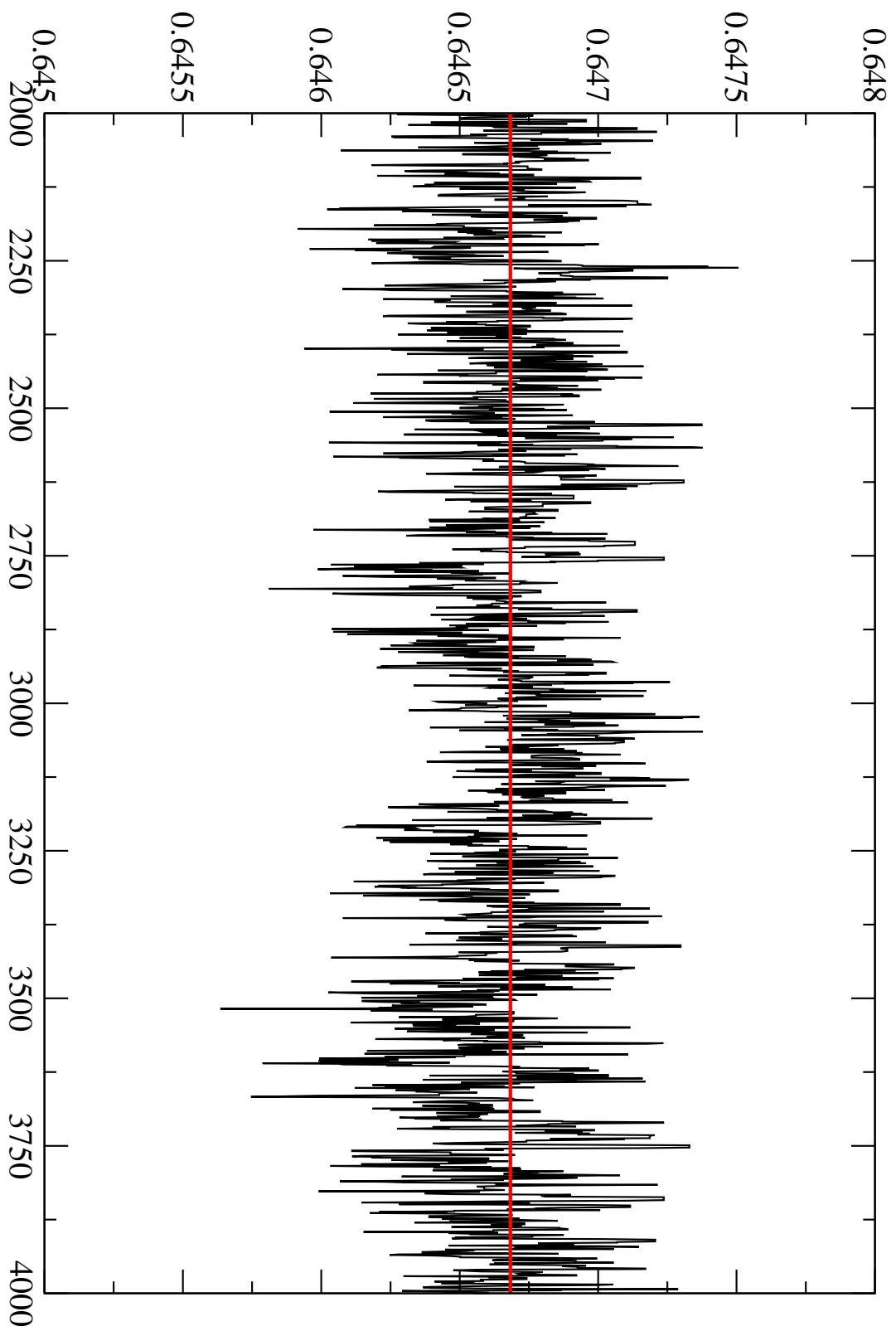
- ‘R’ algorithm with $\Delta\tau = 0.01$
- Check of code: DWF ‘R’ algorithm compared with HMC for $N_f = 2$.



Plaquette Evolution for $N_f = 2$

- $m_{\text{dyn}} = 0.02$



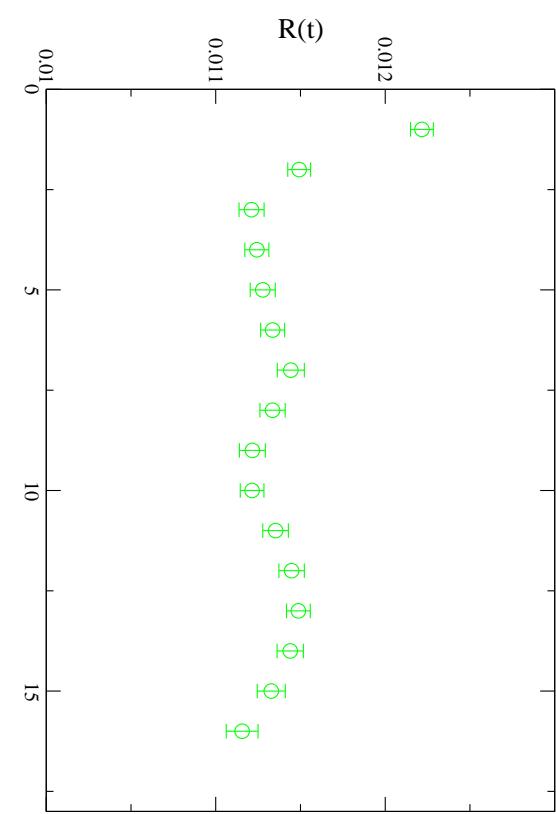
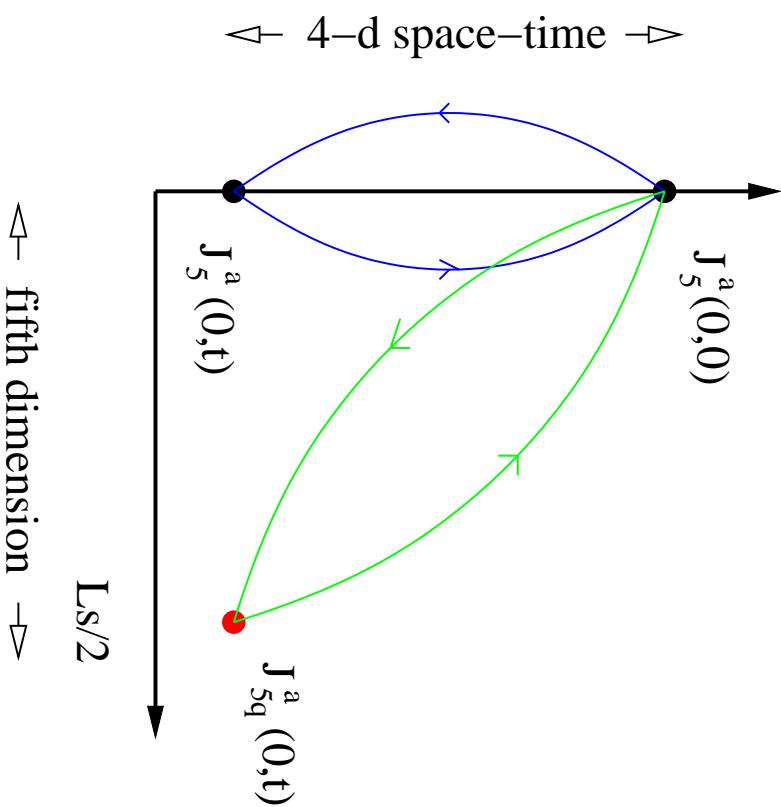


Measuring the residual mass m_{res} for $N_f = 3$

- Simplest use of divergence of axial current: $\Delta_\mu A_\mu^a(x) = 2m_f J_5^a(x) + 2J_{5q}^a(x)$
- Compare pion propagation along $s = 0$ and $L_s - 1$ with propagation to $L_s/2$

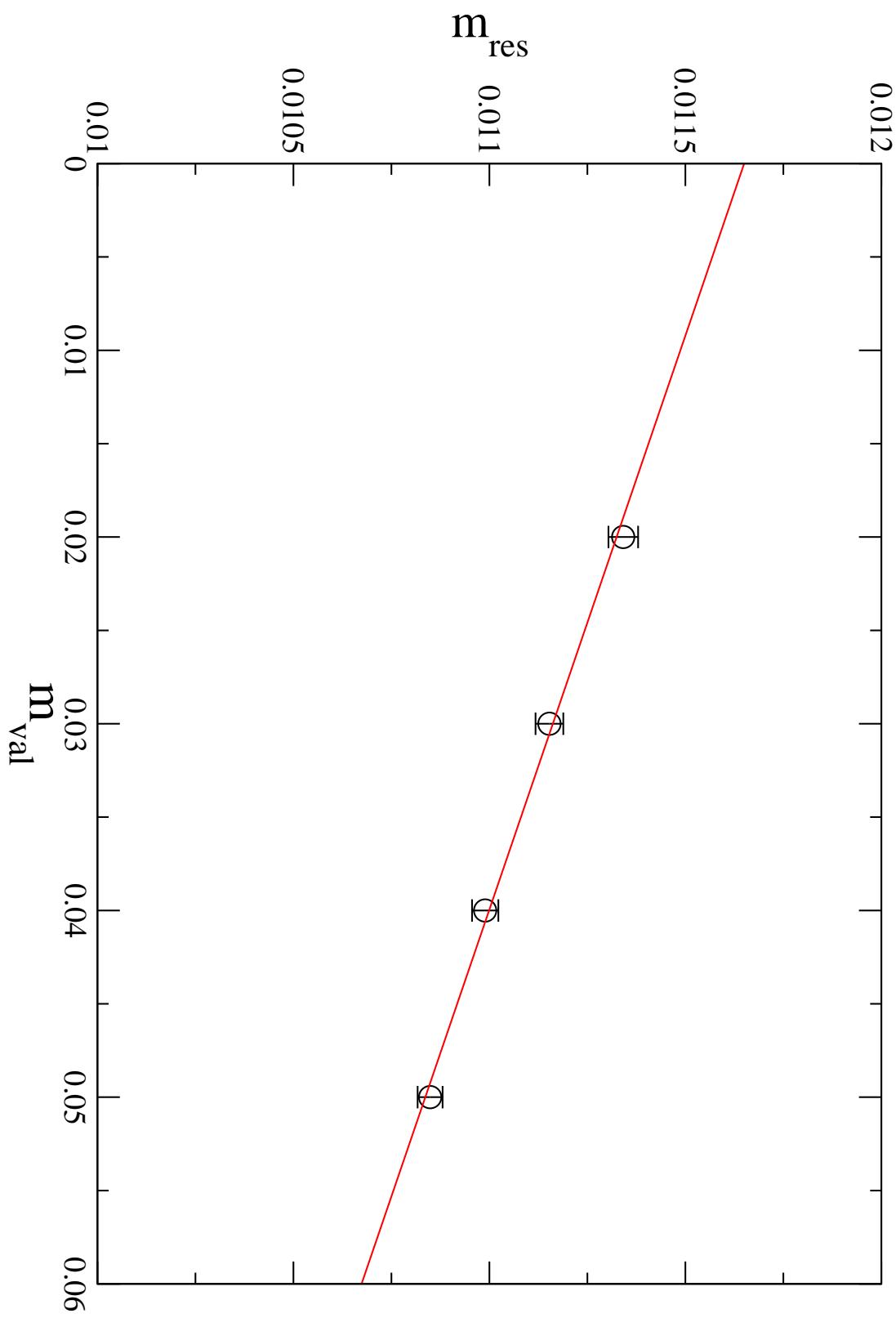
$$R(t) = \frac{\sum_{\vec{x}} \langle J_{5q}^a(\vec{x}, t) J_5^a(0, 0) \rangle}{\sum_{\vec{x}} \langle J_5^a(\vec{x}, t) J_5^a(0, 0) \rangle}$$

$$m_{\text{res}} \equiv \frac{1}{N} \sum_t R(t) \quad \text{if plateau}$$

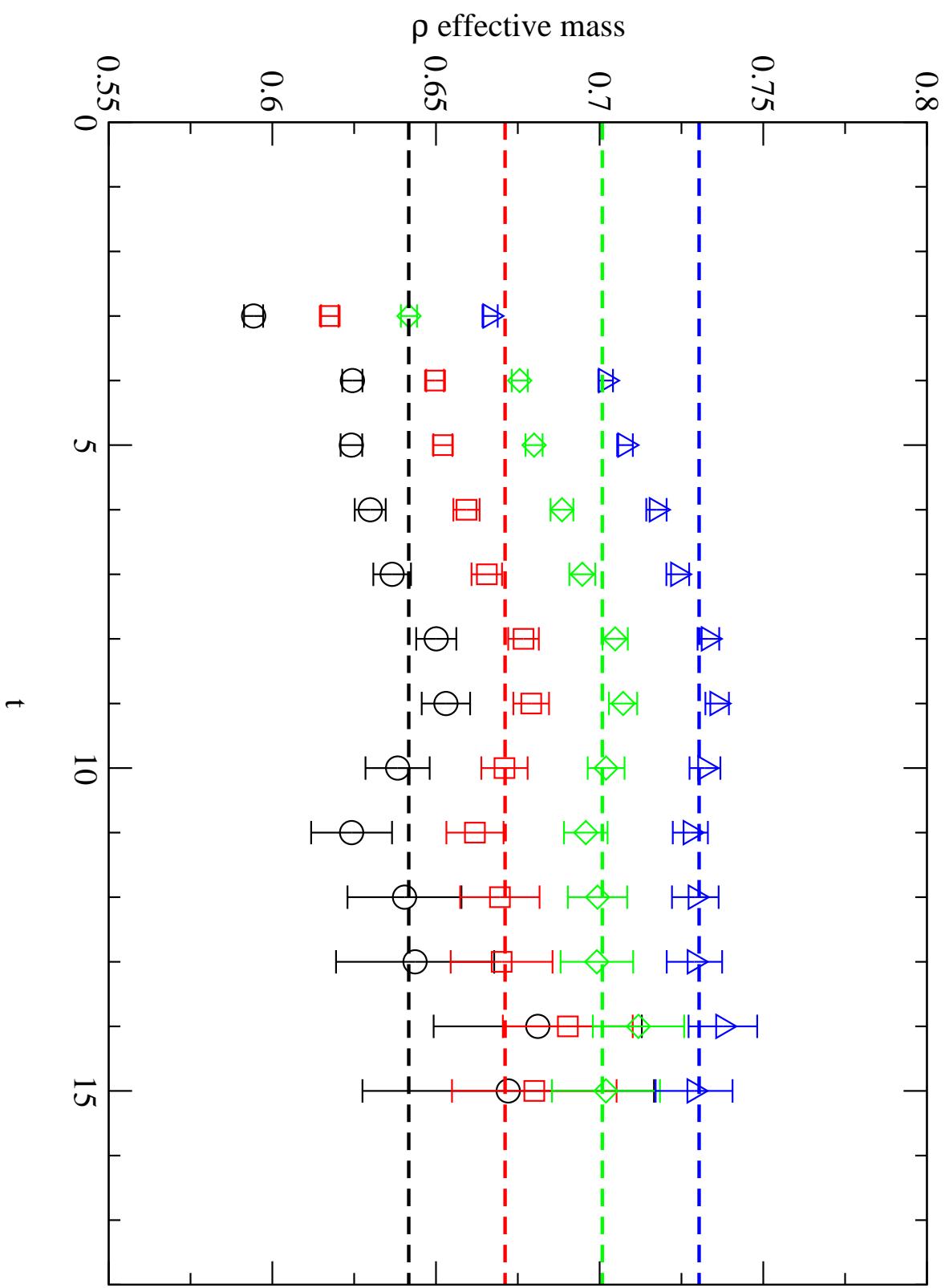


Residual Mass for $N_f = 3$ versus m_{val}

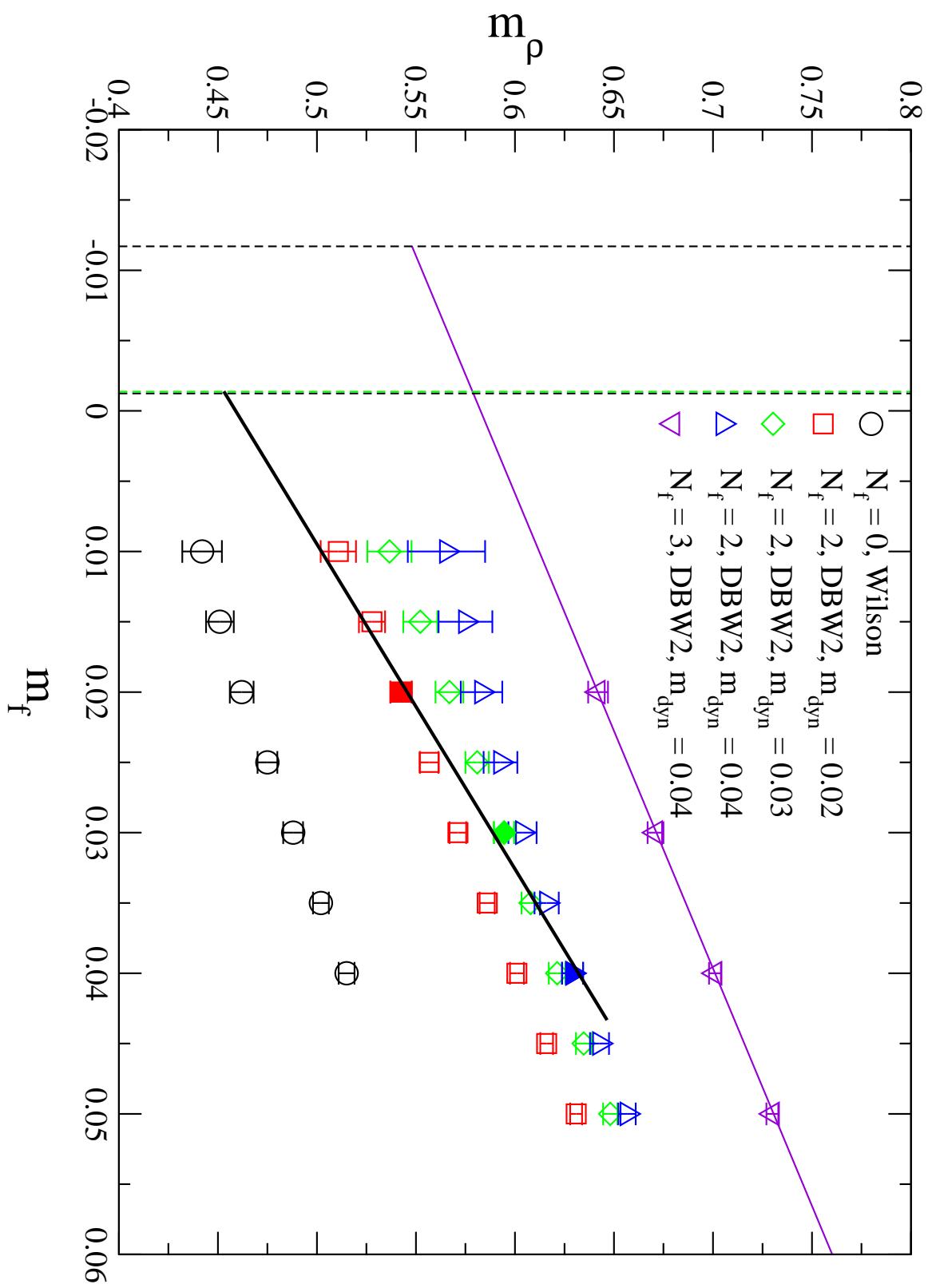
- Extrapolating to $m_f = 0$ gives $m_{\text{res}} = 0.0117(1)$



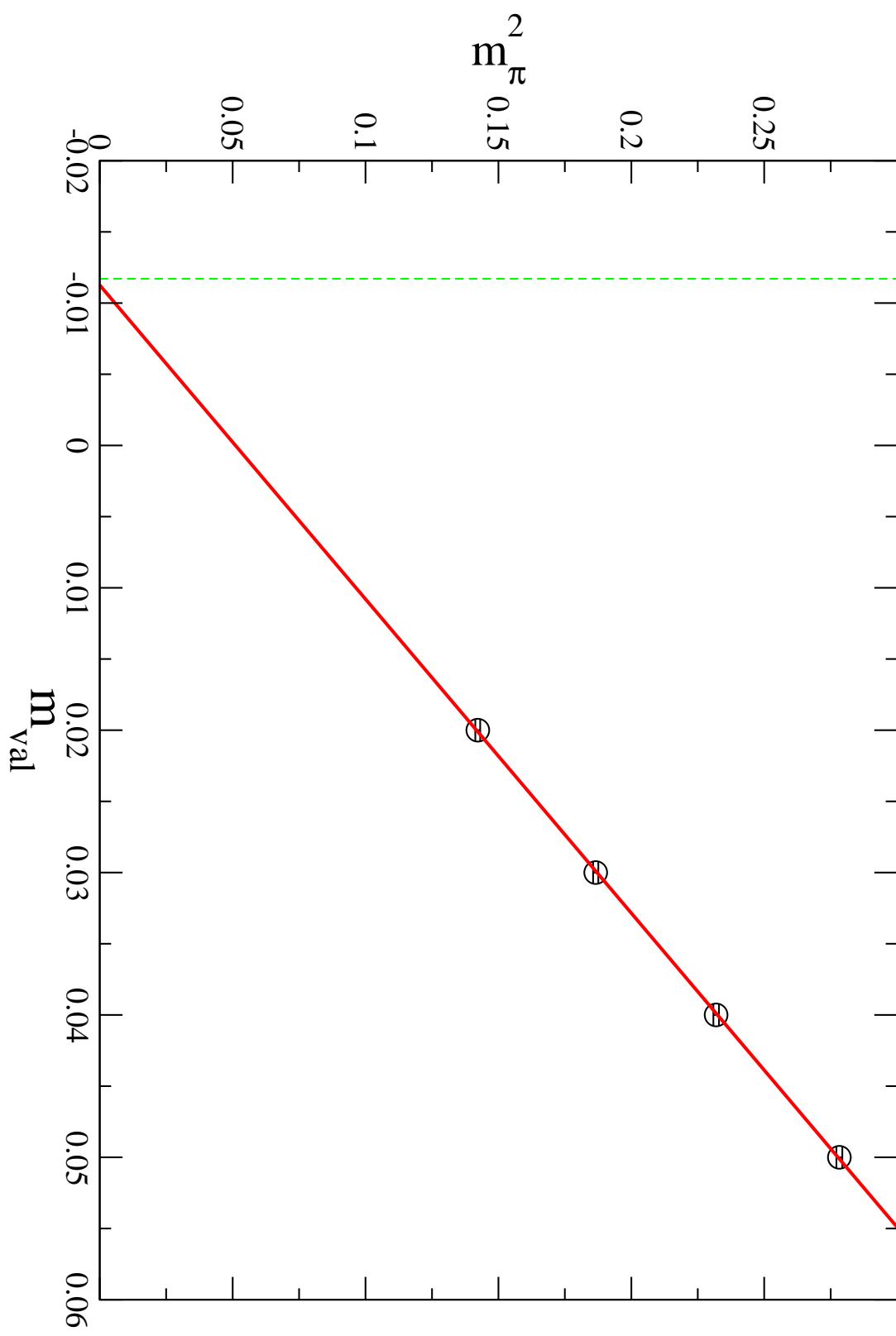
ρ Effective Mass for $N_f = 3$



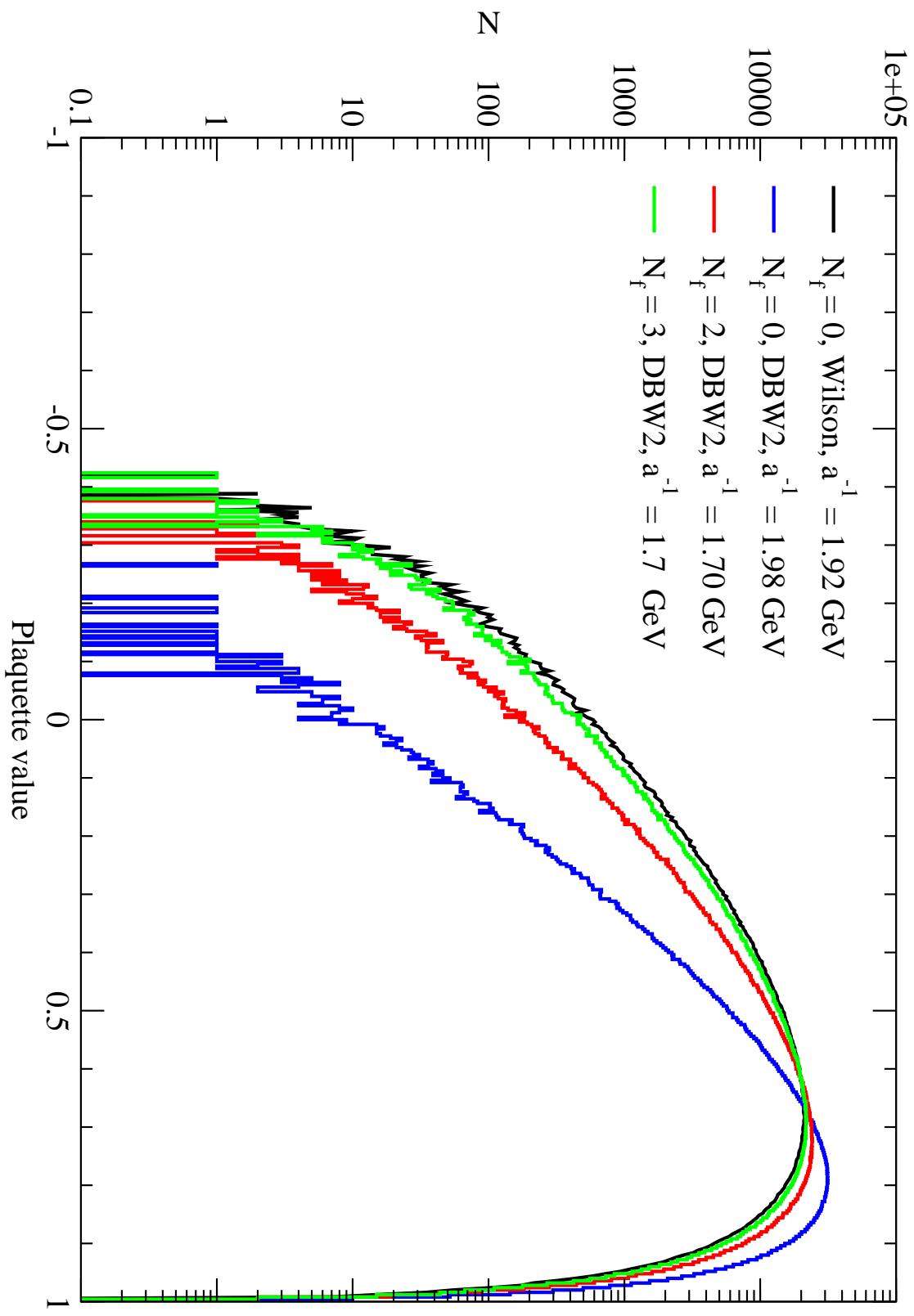
m_ρ versus m_f



m_π^2 versus m_{val} for $N_f = 3$

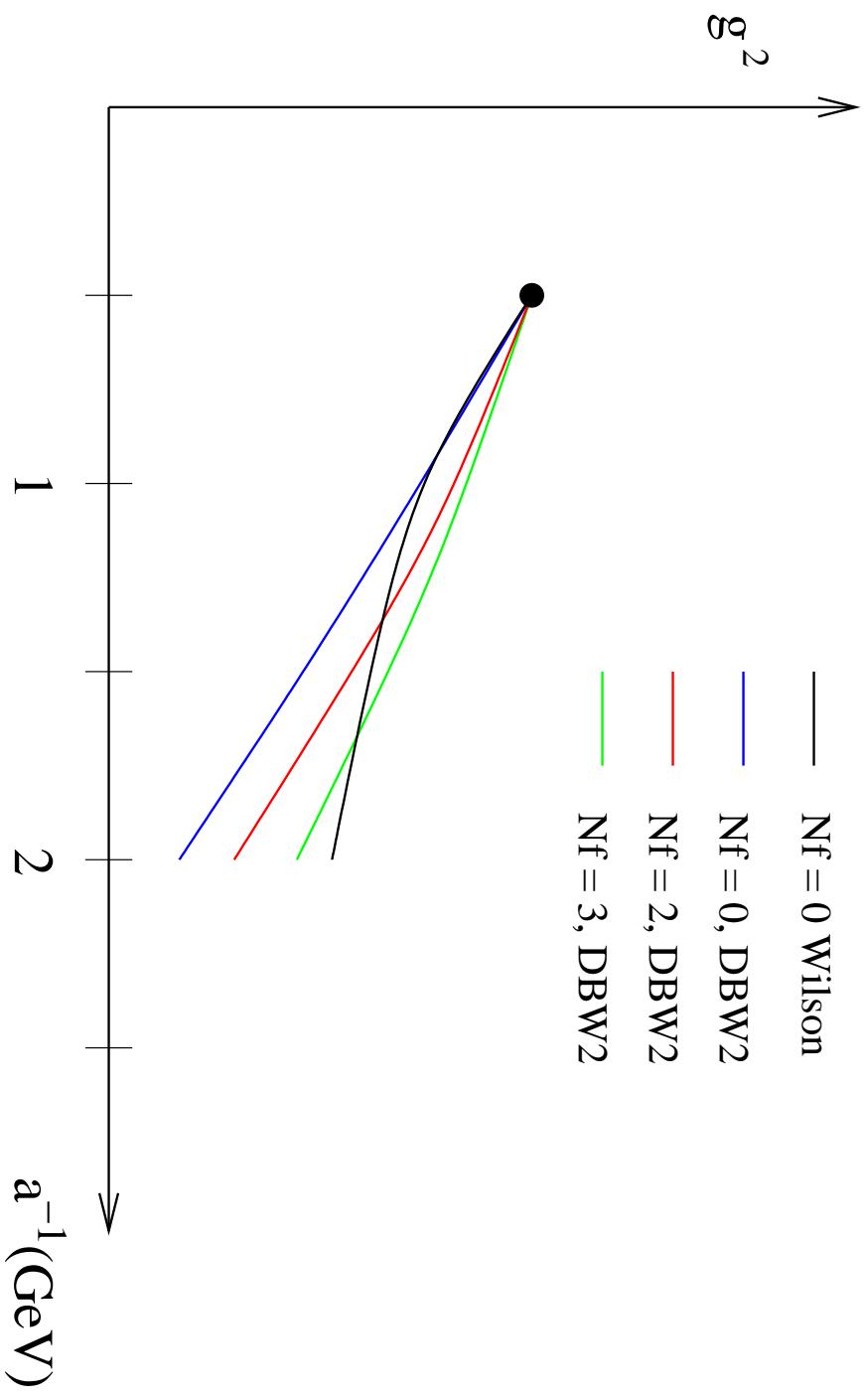


Plaquette Distributions at $a^{-1} \approx 2$ GeV

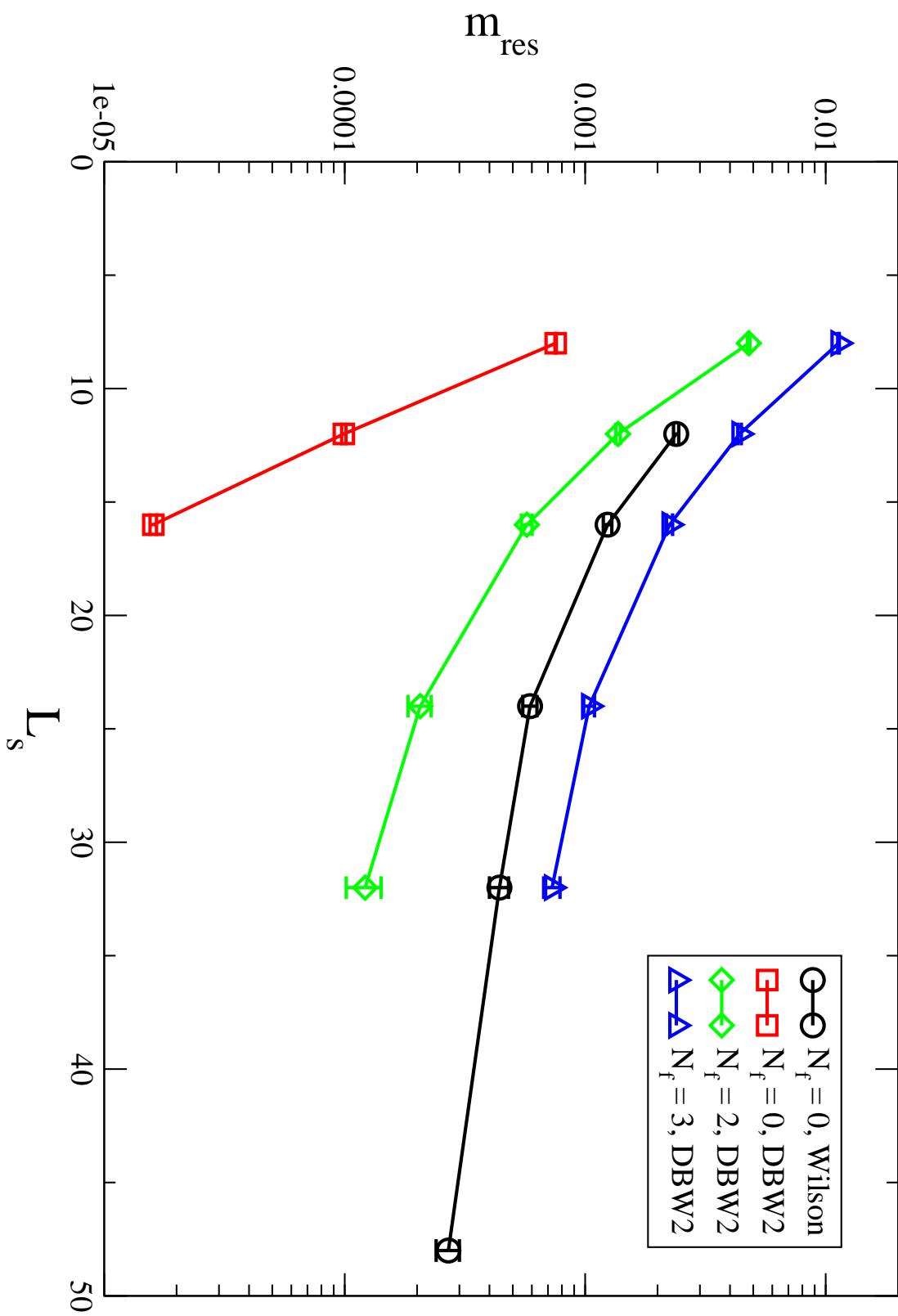


RG Evolution

- DBW2 action changes RG flow, so effective coupling strength at lattice scale is weaker.



m_{res} versus L_s for $N_f = 0, 2$ and 3



Conclusions

- Dynamical DWF simulations only need m_{res} small, not zero.
- Including dynamical fermions roughens gauge fields at the lattice scale, slowing decrease in m_{res} with L_s .
- Looking for ways to improve chiral properties at stronger couplings (see talks by M. Lin and L. Levkova).
- Dynamical DWF simulations with 3 flavors and DBW2 gauge action possible now for a^{-1} above about 2 GeV.
- RHMC code adapted for DWF (Talk by M. Clark)